



# JENN

**Training and Consultancy**  
The path to enlightened education

**SUBJECT/VAK : MATHEMATICS/WISKUNDE**

**GRADE/GRAAD 12**

**2025 WINTER CLASSES/KLASSE**

**SOLUTIONS MANUAL  
OPLOSSINGSHANDLEIDING**

**TOPICS/ONDERWERPE**

- 1. FUNCTIONS AND GRAPHS/FUNKSIES EN GRAFIEKE**
- 2. DIFFERENTIAL CALCULUS (excluding first principles and rules of differentiation)**  
***DIFFERENSIALE REKENING (uitsluitend eerste beginsels en reëls van differentiasie)***
- 3. PROBABILITY (excluding Counting principle)**  
***WAARSKYNNLIKHEID (uitsluitend Telbeginsel)***

# FUNCTIONS AND GRAPHS/FUNKSIES EN GRAFIEKE

## Activity/Aktiwiteit 1

### QUESTION 4/VRAAG 4

4.1.1	decreasing	✓ decreasing (1)
4.1.2	$y = \left(\frac{1}{3}\right)^x$ $x = \left(\frac{1}{3}\right)^y$ $\therefore y = \log_{\frac{1}{3}} x$  <b>OR/OF</b> $y = 3^{-x}$ $x = 3^{-y}$ $\therefore y = -\log_3 x$	✓ swop $x$ and $y$  ✓ answer (2)  <b>OR/OF</b>  ✓ swop $x$ and $y$  ✓ answer (2)
4.1.3	$x > 0; x \in R$	✓ answer (1)
4.1.4	$y = -5$	✓ answer (1)
4.2.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.2.2	$\frac{4}{x-1} + 2 = 0$ $4 = -2x + 2$ $2x = -2$ $x = -1$	✓ let $y = 0$    ✓ $x = -1$ (2)

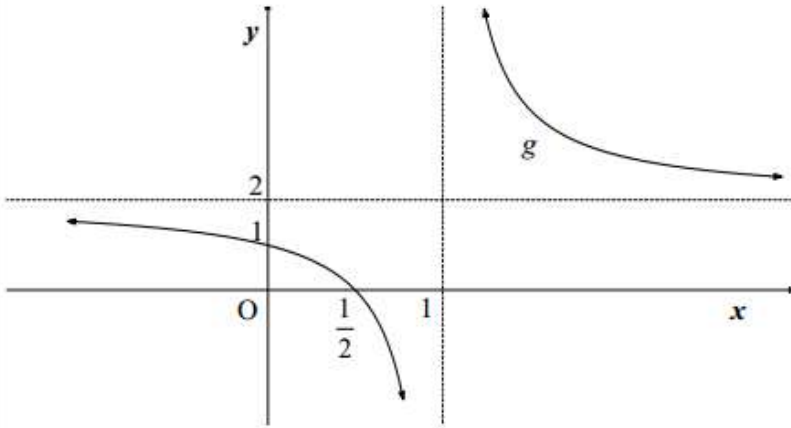
4.2.3		✓ asymptotes ✓ x-intercept ✓ y-intercept ✓ shape (4)
4.2.4	$\frac{4}{x-1} \geq -2$ $\frac{4}{x-1} + 2 \geq 0$ $x \leq -1 \quad \text{or} \quad x > 1$	✓ $x \leq -1$ ✓ $x > 1$ (2)
4.2.5	$y = -x + c$ $2 = -3 + c$ $c = 5$ $y = -x + 5$ <p><b>OR/OF</b></p> $y = -x + c$ $2 = -1 + c$ $c = 3$ $y = -x + 3$ $y = -(x - 2) + 3$ $y = -x + 5$ <p><b>OR/OF</b></p> $y = -(x + p) + q$ $y = -((x - 2) + (-1)) + 2$ $y = -x + 5$	✓ intersection of axes at (3 ; 2) ✓ subst (3 ; 2) and $m = -1$ ✓ $y = -x + 5$ (3) <p><b>OR/OF</b></p> ✓✓ $-(x - 2) + 3$ ✓ $y = -x + 5$ (3) <p><b>OR/OF</b></p> ✓✓ $y = -((x - 2) + (-1)) + 2$ ✓ $y = -x + 5$ (3)
		[18]

### QUESTION 5/VRAAG 5

5.1	T.P(-3;4)	✓ -3 ✓ 4 (2)
5.2	$y \leq 4$ or $y \in (-\infty; 4]$	✓ answer (1)
5.3	$f(x) = g(x)$ $-(x+3)^2 + 4 = x + 5$ $-x^2 - 6x - 9 + 4 = x + 5$ $-x^2 - 7x - 10 = 0$ $x^2 + 7x + 10 = 0$ $(x+5)(x+2) = 0$ $x = -5$ or $x = -2$	✓ equating ✓ $-x^2 - 6x - 9$  ✓ standard form ✓ factors (4)
5.4	The graph must shift more than 2 and less than 5 units to the right $\therefore -5 < c < -2$	✓✓ answer (2)
5.5	$D(x) = f(x) - g(x) = -x^2 - 7x - 10$  Max: $-2x - 7 = 0$ <b>OR/OF</b> $x = \frac{-(-7)}{2(-1)}$  $x = -\frac{7}{2}$  $D\left(-\frac{7}{2}\right) = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10 = 2,25$ $\therefore k = 2,25$ $\therefore h(x) = x + 7,25$	✓ distance  ✓ $-2x - 7 = 0$  ✓ $x = -\frac{7}{2}$  ✓ $k = 2,25$ ✓ $h(x) = x + 7,25$ (5)
		<b>[14]</b>

Die grafiek moet meer as 2 eenhede en minder as 5 eenhede na regs skuif

**Activity/Aktiwiteit 2**  
**QUESTION 4/VRAAG 4**

4.1	$x = 1$ $y = 2$	<ul style="list-style-type: none"> <li>✓ <math>x = 1</math></li> <li>✓ <math>y = 2</math></li> </ul> <p>(2)</p>
4.2		<ul style="list-style-type: none"> <li>✓ x-intercept</li> <li>✓ y-intercept</li> <li>✓ asymptotes</li> <li>✓ shape</li> </ul> <p>(4)</p>
4.3	$x < \frac{1}{2}$ or $x > 1$	<ul style="list-style-type: none"> <li>✓ <math>x &lt; \frac{1}{2}</math></li> <li>✓ <math>x &gt; 1</math></li> </ul> <p>(2)</p>
4.4	$y = -(x - 1) + 2$ $y = -x + 3$  <b>OR/OF</b> $y - 2 = -(x - 1)$ $y = -x + 3$  <b>OR/OF</b> $y = -x + c$ $2 = -(1) + c$ $c = 3$ $\therefore y = -x + 3$	<ul style="list-style-type: none"> <li>✓ substitution of (1 ; 2)</li> <li>✓ answer</li> </ul> <p>(2)</p> <p><b>OR/OF</b></p> <ul style="list-style-type: none"> <li>✓ substitution of (1 ; 2)</li> <li>✓ answer</li> </ul> <p>(2)</p> <p><b>OR/OF</b></p> <ul style="list-style-type: none"> <li>✓ substitution of (1 ; 2)</li> <li>✓ answer</li> </ul> <p>(2)</p>
		<b>[10]</b>

### QUESTION5/VRAAG 5

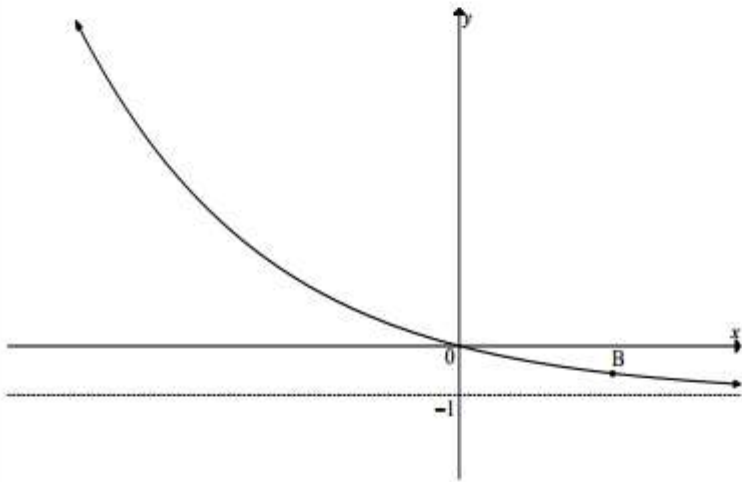
5.1	$P'(2;4)$	$\checkmark x = 2$ $\checkmark y = 4$  (2)
5.2	$f(x) = \log_a x$ $2 = \log_a 4$ $a^2 = 4$ $a = 2$	$\checkmark$ substitute (4 ; 2) $\checkmark a^2 = 4$  (2)
5.3	$y = 2^x$	$\checkmark y = 2^x$  (1)
5.4	$1 = \log_2 x$ $\therefore x = 2$ $T(2; 1)$  RT = 2 units P'T = 3 units  Area of $\Delta RTP' = \frac{1}{2} \cdot RT \cdot TP'$ $= \frac{1}{2} \times 2 \times 3 = 3 \text{ units}^2$	$\checkmark x = 2$  $\checkmark RT = 2 \text{ units}$ $\checkmark P'T = 3 \text{ units}$  $\checkmark$ answer  (4)
		[9]

**QUESTION 6/VRAAG 6**

6.1	$y \geq -4$ or $y \in [-4; \infty)$	✓ answer (1)
6.2	$x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $\therefore E(3; 0)$ and $D(-1; 0)$	✓ = 0  ✓ both x-values ✓ correct identification of coordinates (3)
6.3	$P(0; -3)$ $\therefore m_{PE} = 1$ $\therefore g(x) = x - 3$	✓ $m_{PE} = 1$ ✓ $g(x) = x - 3$ (2)
6.4	$f(x) > g(x)$ $x < 0$ or $x > 3$	✓ $x < 0$ ✓ $x > 3$ (2)
6.5	Distance $= -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$ $D' = -2x + 1 = 0$ or/of $x = -\frac{b}{2a}$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$ $D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $= \frac{25}{4} = 6,25$	✓ $D = -x^2 + x + 6$  ✓ method ✓ $x = \frac{1}{2}$  ✓ substitution  ✓ answer (5)
6.6	$f'(x) = m$ $1 = 2x - 2$ $x = \frac{3}{2}$ Point on $f: \left(\frac{3}{2}; -\frac{15}{4}\right)$ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ $\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2,25$  <b>OR/OF</b> $f(x) = k(x)$ $x^2 - 2x - 3 = x - 3 - n$ $x^2 - 3x + n = 0$ $\Delta = b^2 - 4ac$ $= (-3)^2 - 4(1)(n)$ To touch: $\Delta = 0$ $0 = 9 - 4n$ $4n = 9$ $n = \frac{9}{4}$	✓ $1 = 2x - 2$ ✓ $x = \frac{3}{2}$  ✓ $-\frac{15}{4}$ ✓ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$  ✓ answer (5)  <b>OR/OF</b> ✓ equating ✓ standard form  ✓ substitution into $\Delta$ ✓ $\Delta = 0$  ✓ answer (5)
		<b>[18]</b>

### Activity/Aktiwiteit 3

#### QUESTION/VRAAG 4

4.1	$f(x) = a^x - 1$ $-\frac{5}{9} = a^2 - 1$ $a^2 = \frac{4}{9}$ $a = \frac{2}{3}$	✓ substitution  ✓ answer (2)
4.2	$y > -1$ <b>OR/OF</b> $y \in (-1; \infty)$	✓ answer (1) ✓ answer (1)
4.3		✓ (0 ; 0) ✓ asymptote ✓ decreasing shape (3)
4.4	$\frac{19}{8} = \left(\frac{2}{3}\right)^x - 1$ $\frac{27}{8} = \left(\frac{2}{3}\right)^x$ $\left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-x}$ $\therefore x = -3$ $\therefore C\left(-3; \frac{19}{8}\right)$ $C'\left(\frac{19}{8}; -3\right)$	✓ substitution  ✓ answer  ✓ answer (3)
		[9]



### QUESTION/VRAAG 5

5.1	$p = -1$	✓ $p = -1$ (1)
5.2	$y = x - 3$ $y = 1 - 3$ $y = -2$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">Answer only: <math>\frac{2}{2}</math></div>	✓ substitution ✓ answer (2)
5.3	$1 = \frac{a}{0-1} - 2$ $3 = \frac{a}{-1}$ $a = -3$	✓ substitute (0 ; 1)  ✓ answer (2)
5.4	$\frac{-3}{x-1} - 2 = 0$ $-3 = 2x - 2$ $2x = -1$ $\therefore x = -\frac{1}{2}$ $x \in \left[-\frac{1}{2}; 1\right)$ <b>OR/OF</b> $-\frac{1}{2} \leq x < 1$	✓ x-intercept  ✓✓ answer (3)
5.5	<p><math>f</math> is reflected in the y-axis, then translated 2 units to the right</p> <p><b>OR/OF</b></p> <p><math>f</math> is reflected in the x-axis, then translated 4 units down</p> <p><b>OR/OF</b></p> <p><math>f</math> is reflected about the line <math>x = 1</math></p> <p><b>OR/OF</b></p> <p><math>f</math> is reflected about the line <math>y = -2</math></p>	✓ reflection in y-axis ✓ translation 2 units right (2)  <b>OR/OF</b>  ✓ reflection in x-axis ✓ translation 4 units down (2)  <b>OR/OF</b>  ✓✓ reflection about line $x = 1$ (2)  <b>OR/OF</b>  ✓✓ reflection about line $y = -2$ (2)
		<b>[10]</b>

**5.5**  $f$  is gereflekteer in die y-as, dan 2 eenhede na regs getransleer/geskuif.

**OF**

$f$  is gereflekteer in die x-as, dan 4 eenhede afwaarts getransleer/geskuif

**OF**

$f$  is gereflekteer om die lyn  $x = 1$

**OF**

$f$  is gereflekteer om die lyn  $y = -2$

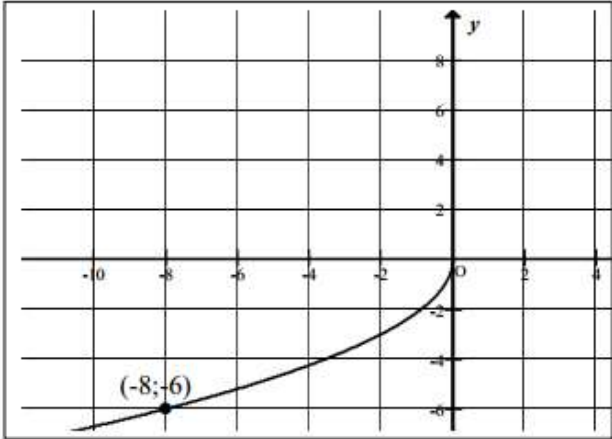
### QUESTION/VRAAG 6

6.1	$x = \frac{-4}{2(-1)} = 2$ <b>OR/OF</b> $f'(x) = -2x + 4 = 0$ $x = 2$ $y = -(2)^2 + 4(2) + 5$ $= 9$ $\therefore B(2; 9)$	✓ method for $x$ ✓ value of $x$ ✓ value for $y$  (3)
6.2	$x^2 - 4x - 5 = 0$ $(x - 5)(x + 1) = 0$ $x = 5$ or $x = -1$ $\therefore A(-1; 0)$ Equation of AC: $\frac{8-0}{3+1} = \frac{y-0}{x+1}$ <b>OR/OF</b> $m = \frac{8-0}{3+1} = 2$ $8x + 8 = 4y$ $y - 0 = 2(x + 1)$ $2x + 2 = y$ $y = 2x + 2$	✓ $A(-1; 0)$  ✓ method for gradient  ✓ substitute point A or C into equation of a straight line (3)
6.3	$EH = -x^2 + 4x + 5 - (2x + 2)$ $EH = -x^2 + 2x + 3$ Max EH at: $-2x + 2 = 0$ <b>OR/OF</b> $x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)}$ $x = 1$ $x = 1$ Max length of EH = $-(1)^2 + 2(1) + 3$ Answer only: $\frac{1}{4}$ $= 4$	✓ difference ✓ $EH = -x^2 + 2x + 3$  ✓ $x$ - value  ✓ length      (4)
6.4	$k(x) = f(x + m)$ $= -x^2 - 2mx - m^2 + 4x + 4m + 5$ $k'(x) = -2x - 2m + 4$ $k'(x) = 2$ $-2x - 2m + 4 = 2$ $-2x = 2m - 2$ $x = 1 - m$  $k(1 - m) = g(1 - m)$ $-(1 - m)^2 - 2m(1 - m) - m^2 + 4(1 - m) + 4m + 5 = 2(1 - m) + 2$ $-1 + 2m - m^2 - 2m + 2m^2 - m^2 + 4 - 4m + 4m + 5 = 2 - 2m + 2$ $-2m - 4 = 0$ $\therefore m = -2$	✓ $k'(x)$ ✓ $k'(x) = 2$  ✓ $x = 1 - m$  ✓ substitution   ✓ answer (5)

<p><b>OR/OF</b></p> $k(x) = f(x + m) = g(x)$ $-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$ $0 = x^2 + (2m - 2)x + m^2 - 4m - 3$ $\Delta = (2m - 2)^2 - 4(1)(m^2 - 4m - 3)$ $\Delta = 4m^2 - 8m + 4 - 4m^2 + 16m + 12$ $\Delta = 8m + 16 = 0$ $\therefore m = -2$ <p><b>OR/OF</b></p> $f(x) = -x^2 + 4x + 5$ $f'(x) = -2x + 4 = 2$ $-2x = -2$ $x = 1$ $f(1) = -(1)^2 + 4(1) + 5$ $\therefore y = 8$ $\therefore \text{translate 2 units to the right}$ $\therefore m = -2 \quad (\text{Transleer/Skuif 2 eenhede na regs})$	<p><b>OR/OF</b></p> <p>✓ equating ✓ simplification</p> <p>✓ substitution into <math>\Delta</math></p> <p>✓ <math>\Delta = 0</math> ✓ answer</p> <p>(5)</p> <p><b>OR/OF</b></p> <p>✓ <math>f'(x) = 2</math></p> <p>✓ x-value</p> <p>✓ y-value ✓ 2 units to the right ✓ answer</p> <p>(5)</p>
	<b>[15]</b>

## Activity/Aktiwiteit 4

### QUESTION 6

6.1	$f(x) = ax^2$ $-8 = a(-6)^2$ $-8 = 36a$ $a = -\frac{8}{36}$ <b>OR</b> $a = -\frac{2}{9}$	✓ substitution  ✓ answer (2)
6.2	$f(x): y = -\frac{2}{9}x^2$ $x = -\frac{2}{9}y^2$ $9x = -2y^2$ $-\frac{9x}{2} = y^2$ $y = \pm\sqrt{\frac{-9x}{2}}$ , since $y \leq 0$ $y = -\sqrt{\frac{-9x}{2}}$ <b>OR</b> $y = -3\sqrt{\frac{-x}{2}}$	✓ swop x and y  ✓ $y^2 = -\frac{9x}{2}$ or $y = \pm\sqrt{\frac{-9x}{2}}$  ✓ $y = -\sqrt{\frac{-9x}{2}}$ (3)
6.3	$y \leq 0$  <b>OR</b> $y \in (-\infty ; 0]$	✓ answer (1)
6.4		✓ shape (third quadrant) (concave upward) ✓ Any point other than (0 ; 0) that lies on the graph  Point corresponding from original graph will be (-8 ; -6) (2)

6.5	$y = -f^{-1}(x)$ $= \sqrt{\frac{-9x}{2}}$ <p><b>OR</b></p> $y = -\frac{2}{9}x^2$ <p>Reflection in <math>y = x</math>: <math>x = -\frac{2}{9}y^2</math></p> $-\frac{9}{2}x = y^2$ $y = -\sqrt{-\frac{9x}{2}}$ <p>Reflection about <math>y</math>-axis: <math>y = \sqrt{-\frac{9x}{2}}</math></p>	$\checkmark \checkmark -f^{-1}(x)$ $\checkmark \text{ answer}$ <p>(3)</p> $\checkmark x = -\frac{2}{9}y^2$ $\checkmark y = -\sqrt{-\frac{9x}{2}}$ $\checkmark y = \sqrt{-\frac{9x}{2}}$ <p>(3)</p> <p><b>[11]</b></p>
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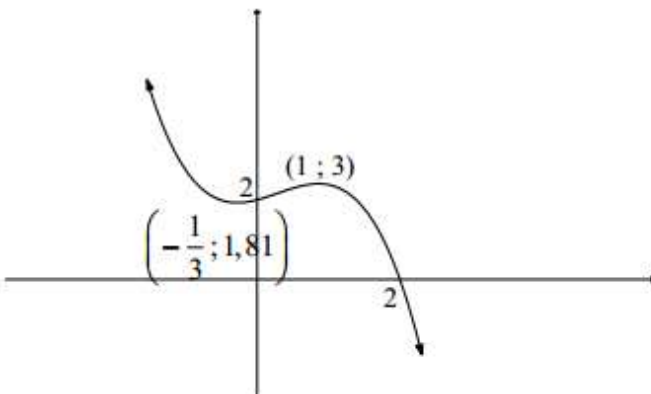
# DIFFERENTIAL CALCULUS/DIFFERENTIALE REKENING

## Activity/Aktiweiteit 1

### QUESTION 8/VRAAG 8

<p>8.1</p> $f'(x) = mx^2 + nx + k$ $f'(x) = m\left(x + \frac{1}{3}\right)(x - 1)$ $1 = m\left(0 + \frac{1}{3}\right)(0 - 1)$ $1 = -\frac{1}{3}m$ $\therefore m = -3$ $f'(x) = -3\left(x + \frac{1}{3}\right)(x - 1)$ $f'(x) = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$ $f'(x) = -3x^2 + 2x + 1$ $\therefore n = 2$ $\therefore k = 1$ <p><b>OR/OF</b></p> $k = 1$ $0 = m + n + 1 \quad \text{and} \quad \frac{1}{9}m - \frac{1}{3}n + 1 = 0$ $m + n = -1 \quad (1)$ $m - 3n = -9 \quad (2)$ $(1) - (2)$ $4n = 8$ $\therefore n = 2$ $m + 2 = -1$ $\therefore m = -3$	$\checkmark \text{ substitution of } \left(-\frac{1}{3}; 0\right)$ $\text{and } (1; 0)$ $\checkmark \text{ substitution of } (0; 1)$ $\checkmark m = -3$ $\checkmark f'(x) = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$ $\checkmark n = 2$ $\checkmark k = 1 \quad (6)$ <p><b>OR/OF</b></p> $\checkmark k = 1$ $\checkmark m + n = -1$ $\checkmark m - 3n = -9$ $\checkmark 4n = 8$ $\checkmark n = 2$ $\checkmark m = -3 \quad (6)$
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8.2.1	$f(x) = -x^3 + x^2 + x + 2$ $f\left(-\frac{1}{3}\right) = \frac{49}{27} = 1,81$ T.P $\left(-\frac{1}{3}; \frac{49}{27}\right)$ $f(1) = 3$ T.P(1; 3)	✓ x-coordinates of the TP  ✓ T.P $\left(-\frac{1}{3}; \frac{49}{27}\right)$  ✓ T.P(1; 3) (3)
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8.2.2	$f(x) = -x^3 + x^2 + x + 2$ $-x^3 + x^2 + x + 2 = 0$ $(x-2)(-x^2 - x - 1) = 0$ $x = 2 \text{ or no solution} \quad / \text{ (Geen oplossing)}$ 	$\checkmark x = 2$  $\checkmark$ one $x$ -intercept $\checkmark$ two turning points $\checkmark$ $y$ -intercept $\checkmark$ shape: neg cubic (5)
8.3.1	$a = \frac{-\frac{1}{3} + 1}{2}$ $= \frac{1}{3}$ <b>OR/OF</b> $f'(x) = -3x^2 + 2x + 1$ $f''(x) = -6x + 2$ $f''(a) = -6a + 2 = 0$ $-6a = -2$ $a = \frac{1}{3}$	$\checkmark$ answer (1)  <b>OR/OF</b>  $\checkmark$ answer (1)
8.3.2	$b < \frac{4}{3} \text{ units}$	$\checkmark \frac{4}{3}$ $\checkmark b < \frac{4}{3}$ (2)
		[17]



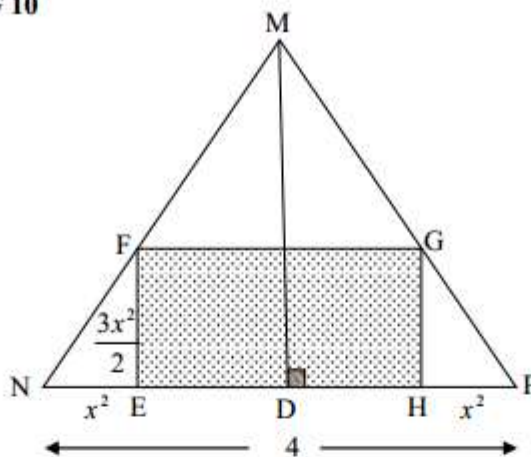
## Activity/Aktiwiteit 2

### QUESTION9/VRAAG 9

9.1	$f(x) = ax^3 + bx^2 + cx - 5$ $-5 = a(0+1)^2(0-5)$ $-5 = -5a$ $a = 1$ $f(x) = (x+1)(x+1)(x-5)$ $f(x) = (x^2 + 2x + 1)(x-5)$ $f(x) = x^3 - 3x^2 - 9x - 5$ $\therefore b = -3$ and $c = -9$	<ul style="list-style-type: none"> <li>✓ substitution of <math>x</math>-intercepts</li> <li>✓ simplification</li> </ul> <ul style="list-style-type: none"> <li>✓ simplification</li> </ul>
9.2	$f(x) = x^3 - 3x^2 - 9x - 5$ $f'(x) = 3x^2 - 6x - 9$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ <b>Minimum</b> value at $x = 3$	<ul style="list-style-type: none"> <li>✓ <math>f'(x) = 3x^2 - 6x - 9</math></li> <li>✓ <math>f'(x) = 0</math></li> <li>✓ factors</li> <li>✓ <math>x = 3</math></li> </ul>
9.3	$f''(x) \cdot f'(x) > 0$ Point of inflection: $x = 1$ <b>Buigpunt/infleksiepunt</b> $x < 1$ ; $x \neq -1$ or $x > 3$	<ul style="list-style-type: none"> <li>✓ <math>x = 1</math></li> <li>✓ <math>x &lt; 1</math>; <math>x \neq -1</math></li> <li>✓ <math>x &gt; 3</math></li> </ul>
9.4	$-32 < -t < -5$ $5 < t < 32$  <b>OR/OF</b> Shift up more than 5 units and less than 32 units $\therefore 5 < t < 32$	<ul style="list-style-type: none"> <li>✓ <math>-32</math></li> <li>✓ <math>-32 &lt; -t &lt; -5</math></li> <li>✓ <math>5 &lt; t &lt; 32</math></li> </ul> <b>OR/OF</b> <ul style="list-style-type: none"> <li>✓ more than 5 units</li> <li>✓ less than 32 units</li> <li>✓ <math>5 &lt; t &lt; 32</math></li> </ul>
		<b>[13]</b>

↪ Skuif meer as 5 eenhede en minder as 32 eenhede op

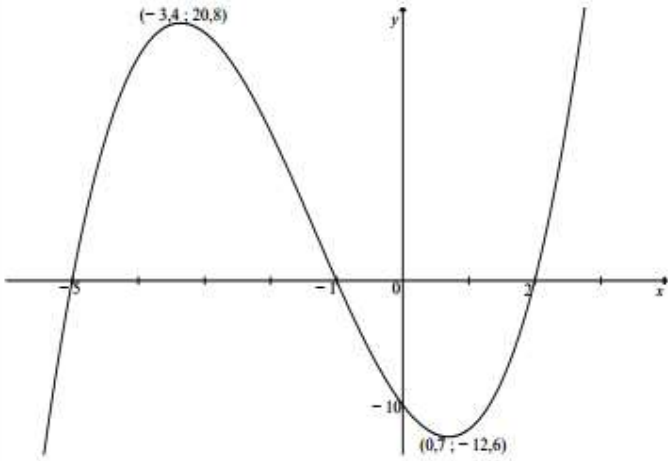
QUESTION 10/VRAAG 10



10.1	$\frac{NE}{EF} = \frac{2}{3} = \frac{x^2}{b}$ $3x^2 = 2b$ $\therefore b = \frac{3x^2}{2}$ $EH = 4 - 2x^2$ $\text{Area EFGH} = (4 - 2x^2) \left( \frac{3x^2}{2} \right)$ $A(x) = 6x^2 - 3x^4$ <p><b>OR/OF</b></p> <p>In <math>\triangle DMP</math>: <math>\tan P = \frac{3}{2}</math></p> <p>In <math>\triangle HGP</math>: <math>\tan P = \frac{GH}{x^2}</math></p> $\frac{GH}{x^2} = \frac{3}{2}$ $\therefore b = \frac{3x^2}{2}$ $EH = 4 - 2x^2$ $\text{Area EFGH} = (4 - 2x^2) \left( \frac{3x^2}{2} \right)$ $A(x) = 6x^2 - 3x^4$	$\checkmark \frac{NE}{EF} = \frac{2}{3} = \frac{x^2}{b}$ $\checkmark \therefore b = \frac{3x^2}{2}$ $\checkmark EH = 4 - 2x^2$ $\checkmark (4 - 2x^2) \left( \frac{3x^2}{2} \right)$ <p style="text-align: right;">(4)</p> <p><b>OR/OF</b></p> $\checkmark \frac{GH}{x^2} = \frac{3}{2}$ $\checkmark \therefore b = \frac{3x^2}{2}$ $\checkmark EH = 4 - 2x^2$ $\checkmark (4 - 2x^2) \left( \frac{3x^2}{2} \right)$ <p style="text-align: right;">(4)</p>
10.2	$A(x) = 6x^2 - 3x^4$ $A'(x) = 12x - 12x^3 = 0$ $12x(1 - x^2) = 0$ $\therefore x \neq 0 \text{ or } x = -1 \text{ or } x = 1$ $\therefore \text{max area: } A(1) = 6(1)^2 - 3(1)^4 = 3 \text{ cm}^2$	$\checkmark 12x - 12x^3 = 0$ $\checkmark \text{values of } x$ $\checkmark \text{correct substitution}$ $\checkmark \text{answer}$ <p style="text-align: right;">(4)</p>
		<b>[8]</b>

### Activity/Aktiwiteit 3

#### QUESTION 8/VRAAG 8

8.1	$y = -10$	✓ answer (1)
8.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f(2) = 2^3 + 4(2)^2 - 7(2) - 10 = 0$	✓ substitution of $x = 2$ ✓ $f(2) = 0$ (2)
8.3	$f(x) = (x-2)(x^2 + 6x + 5)$ $f(x) = (x-2)(x+5)(x+1)$	✓ $(x-2)$ ✓ $(x+5)$ ✓ $(x+1)$ (3)
8.4		✓ x- intercepts ✓ y-intercept ✓ sketching the graph with turning points in 2 <sup>nd</sup> and 4 <sup>th</sup> quadrant (3)
8.5.1	$x \in (-3,4 ; 0,7)$ <b>OR/OF</b> $-3,4 < x < 0,7$	✓ ✓ $x \in (-3,4 ; 0,7)$ (2)
8.5.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f'(x) = 3x^2 + 8x - 7$ $f''(x) = 6x + 8 = 0$ $\therefore x = -\frac{8}{6} = -\frac{4}{3} = -1,33$  <b>OR/OF</b> $\frac{-3,4 + 0,7}{2} = -1,35 = -1,35$	✓ $f''(x) = 6x + 8$ ✓ answer (2)  <b>OR/OF</b> ✓ substitution ✓ answer (2)
8.5.3	$x \leq -3,4$ or $-1,33 \leq x \leq 0,7$ <b>OR/OF</b> $x \in (-\infty ; -3,4] \cup [-1,33 ; 0,7]$	✓ $x \leq -3,4$ (A) ✓ ✓ $-1,33 \leq x \leq 0,7$ (A 0,7) (3)
		[16]

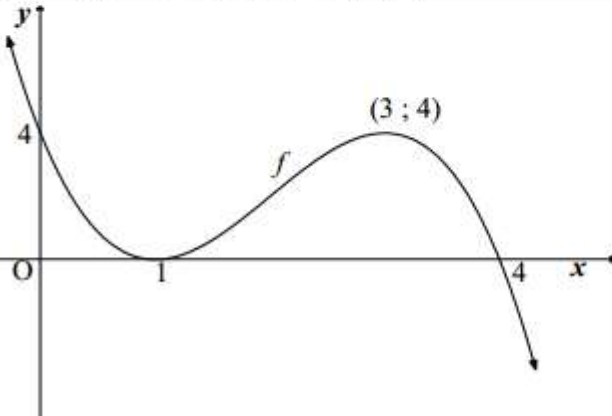
**QUESTION 9/VRAAG 9**

Omtrek van die vierkant

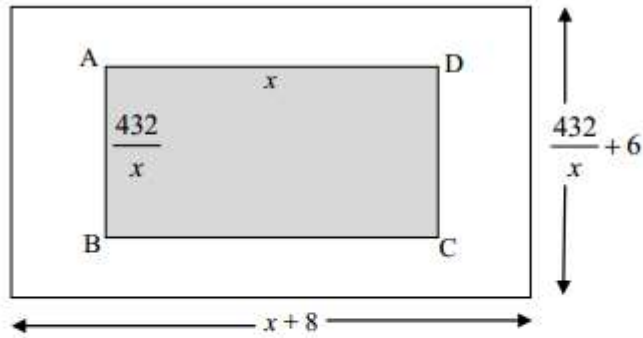
9.1	<p>Perimeter of the square = <math>12 - 6x</math></p> <p>Side length of square = <math>\frac{12 - 6x}{4} = \frac{6 - 3x}{2} = 3 - \frac{3}{2}x</math></p>	<p>✓ <math>12 - 6x</math></p> <p>✓ answer (2)</p>
9.2	<p>Sylengte van 'n vierkant</p> <p><math>V = \left(\frac{6-3x}{2}\right)^2 (4x)</math></p> <p><math>= \left(\frac{36 - 36x + 9x^2}{4}\right)(4x)</math></p> <p><math>= 36x - 36x^2 + 9x^3</math></p> <p><math>V(x) = 36x - 36x^2 + 9x^3</math></p> <p><math>V'(x) = 36 - 72x + 27x^2</math></p> <p><math>36 - 72x + 27x^2 = 0</math></p> <p><math>9x^2 - 24x + 12 = 0</math></p> <p><math>3x^2 - 8x + 4 = 0</math></p> <p><math>(3x - 2)(x - 2) = 0</math></p> <p><math>x = \frac{2}{3}</math> or <math>x = 2</math></p> <p><math>V\left(\frac{2}{3}\right) = 36\left(\frac{2}{3}\right) - 36\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right)^3</math></p> <p><math>= \frac{32}{3} \text{ m}^3 = 10,67 \text{ m}^3</math></p>	<p>✓ <math>\left(\frac{6-3x}{2}\right)^2 (4x)</math></p> <p>✓ <math>\left(\frac{36 - 36x + 9x^2}{4}\right)</math></p> <p>✓ <math>36x - 36x^2 + 9x^3</math></p> <p>✓ <math>V'</math></p> <p>✓ <math>V' = 0</math></p> <p>✓ values</p> <p>✓ answer (7)</p>
		<b>[9]</b>

### Activity/Aktiwiteit 4

#### QUESTION 8/VRAAG 8

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3 \text{ or } x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: $(3; 4)$ and $(1; 0)$	$\checkmark$ $f'(x) = -3x^2 + 12x - 9$ $\checkmark f'(x) = 0$  $\checkmark$ both x-values  $\checkmark$ both y-values (4)
8.2		$\checkmark$ y-intercept $\checkmark$ both x-intercepts $\checkmark$ both turning points $\checkmark$ shape (4)
8.3	$0 < k < 4$ or/of $k \in (0; 4)$	$\checkmark \checkmark k$ between y-values of turning points (2)
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at $(2; 2)$  $f'(2) = 3$ $\therefore y - 2 = 3(x - 2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$  <b>OR/OF</b> <b>Buigpunt/infleksiepunt</b> Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at $(2; 2)$  $f'(2) = 3$ $\therefore y - 2 = 3(x - 2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$	$\checkmark f''(x) = -6x + 12$ $\checkmark f''(x) = 0$ $\checkmark$ x-value $\checkmark$ y-value $\checkmark$ gradient at x-value  $\checkmark$ equation of tangent (6)  <b>OR/OF</b> $\checkmark \checkmark \frac{3+1}{2}$ $\checkmark$ x-value $\checkmark$ y-value  $\checkmark$ gradient at x-value $\checkmark$ equation of tangent (6)
8.5	$\tan \theta = 3$ $\therefore \theta = 71,57^\circ$	$\checkmark$ gradient of g $\checkmark$ answer (2)
		[18]

**QUESTION 9/VRAAG 9**



9.1	$432 = xb$ $\therefore b = \frac{432}{x}$ $A(x) = (x+8)\left(\frac{432}{x} + 6\right)$ $A(x) = 432 + 6x + \frac{3456}{x} + 480$ $A(x) = \frac{3456}{x} + 6x + 480$	$\checkmark b = \frac{432}{x}$ $\checkmark (x+8)$ $\checkmark \left(\frac{432}{x} + 6\right)$	(3)
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A'(x) = -\frac{3456}{x^2} + 6$ $-\frac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24 \text{ cm}$	$\checkmark 3456x^{-1} + 6x + 480$ $\checkmark A'(x) = -\frac{3456}{x^2} + 6$  $\checkmark \text{answer}$	(3)
			[6]

### Activity/Aktiwiteit 5

8.2	$f(x) = x^3 - 4x^2 + 2x + 3$ $f'(x) = 3x^2 - 8x + 2$ $m = f'(2) = 3(2)^2 - 8(2) + 2$ $m = -2$ $y = f(2) = (2)^3 - 4(2)^2 + 2(2) + 3$ $y = -1$ $y + 1 = -2(x - 2)$ $y = -2x + 3$	$\checkmark m = -2$  $\checkmark y = -1$  $\checkmark \text{answer}$
8.3.1	$f(x) = -6x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-6(x+h)^2 + 6x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-6x^2 - 12xh - 6h^2 + 6x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-12xh - 6h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-12x - 6h)}{h}$ $= \lim_{h \rightarrow 0} (-12x - 6h)$ $= -12x$	$\checkmark \text{substitution}$  $\checkmark f(x+h) = -6x^2 - 12xh - 6h^2$  $\checkmark \text{simplification}$  $\checkmark \text{common factor}$  $\checkmark \text{answer}$



	<p><b>OR/OF</b></p> $f(x) = -6x^2$ $f(x+h) = -6x^2 - 12xh - 6h^2$ $f(x+h) - f(x) = -6x^2 - 12xh - 6h^2 + 6x^2 = -12xh - 6h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-12xh - 6h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-12x - 6h)}{h}$ $= \lim_{h \rightarrow 0} (-12x - 6h)$ $= -12x$	<p><b>OR/OF</b></p> $\checkmark f(x+h) = -6x^2 - 12xh - 6h^2$ $\checkmark \text{simplification}$ $\checkmark \text{substitution}$ $\checkmark \text{common factor}$ $\checkmark \text{answer}$ <p style="text-align: right;">(5)</p>
8.3.2	$x \geq 0$ <p><b>OR/OF</b></p> $x \leq 0$	$\checkmark \text{answer}$ <p style="text-align: right;">(1)</p> $\checkmark \text{answer}$ <p style="text-align: right;">(1)</p>
8.3.3	$y = -6x^2$ $x = -6y^2$ $y^2 = \frac{-1}{6}x$ $y = \pm \sqrt{-\frac{1}{6}x}$ $\therefore y = -\sqrt{-\frac{1}{6}x} \quad ; \quad x \leq 0$	$\checkmark \text{swopping } x \text{ and } y$ $\checkmark y = \pm \sqrt{-\frac{1}{6}x}$ $\checkmark \text{answer}$ <p style="text-align: right;">(3)</p>
		<b>[18]</b>



**QUESTION/VRAAG 9**

9.1	$1 < x < \frac{5}{2}$	✓✓ answer (2)
9.2	$(1; 0)$ and $\left(\frac{5}{2}; 0\right)$	✓ $x = 1$ ✓ $x = \frac{5}{2}$ (2)
9.3	$\frac{\frac{5}{2} + 1}{2}$ $= \frac{7}{4}$ Concave up for $x > \frac{7}{4}$ (Konkaaf op vir:)	✓ method  ✓ answer (2)
9.4	$-9 < k < -8$	✓✓ answer (2)
		<b>[8]</b>

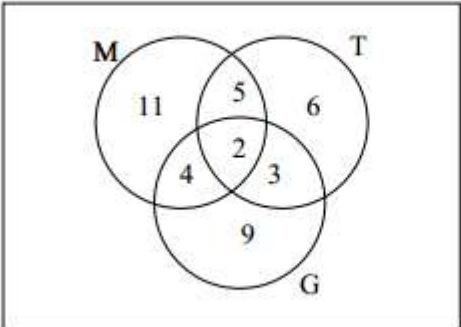
**QUESTION/VRAAG 10**

10.1	$-6t + 18 = 0$ $18 = 6t$ $3 = t$ $s'(3) = -3(3)^2 + 18(3) = 27$	✓ $= 0$  ✓ value of $t$ ✓ answer (3)
10.2	$-3t^2 + 18t = 0$ $-3t(t - 6) = 0$ $t = 0$ or $t = 6$  $s(t) = at^3 + bt^2$ $s'(t) = 3at^2 + 2bt$ $3a = -3$ and $2b = 18$ $a = -1$ $b = 9$  $s(t) = -1t^3 + 9t^2$ $s(6) = -(6)^3 + 9(6)^2$ $s(6) = 108$	✓ factors ✓ values   ✓ $a$ and $b$  ✓ substitution ✓ answer (5)
		<b>[8]</b>

## PROBABILITY/WAARSKYNLIKHEID

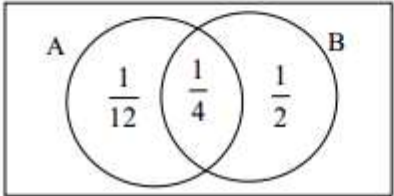
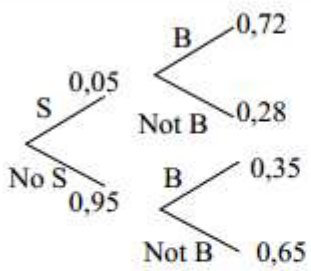
### Activity/Aktiwiteit 1

#### QUESTION/VRAAG 11

11.1		<p>✓2</p> <p>✓3 ; 4 ; 5</p> <p>✓6 ; 9 ; 11</p> <p style="text-align: right;">(3)</p>
11.2	$P(\text{at least two subjects}) = \frac{4 + 2 + 5 + 3}{40} = \frac{14}{40} = \frac{7}{20} = 0,35$	<p>✓ (4 + 2 + 5 + 3) or 14 in numerator</p> <p>✓ 40 in denominator</p> <p style="text-align: right;">(2)</p>
11.3	$P(M) \times P(T) = \frac{22}{40} \times \frac{16}{40} = \frac{11}{50} = 0,22$ $P(M \text{ and } T) = \frac{7}{40} = 0,175$ $P(M) \times P(T) \neq P(M \text{ and } T)$ <p>Events M and T are not independent</p> <p>(Gebeurtenisse M en T is nie onafhanklik nie)</p>	<p>✓ <math>\frac{22}{40} \times \frac{16}{40}</math></p> <p>✓ <math>\frac{11}{50}</math></p> <p>✓ <math>\frac{7}{40}</math></p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
		<b>[9]</b>

## Activity/Aktiwiteit 2

### QUESTION 10/VRAAG 10

10.1.1	$P(A \text{ and } B) = P(A) \times P(B)$ $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$	$\checkmark \frac{1}{3} \times \frac{3}{4}$ $\checkmark \frac{1}{4}$ (2)
10.1.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ <p><b>OR/OF</b></p>  $P(A \text{ or } B) = \frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$	$\checkmark$ substitution $\checkmark$ answer (2)
10.2.1		$\checkmark$ branch 1 with probabilities $\checkmark$ branch 2 with probabilities $\checkmark$ branch 3 with probabilities (3)
10.2.2	$P(\text{NOT below } 0^\circ)$ $= P(S; \text{NOT below } 0^\circ) + P(NS; \text{NOT below } 0^\circ)$ $= 0,05 \times 0,28 + 0,95 \times 0,65$ $= 0,6315$	$\checkmark$ value of $P(S; \text{NOT below } 0^\circ)$ $\checkmark$ value of $P(NS; \text{NOT below } 0^\circ)$ $\checkmark$ answer (3)

### Activity/Aktiwiteit 3

#### QUESTION 11/VRAAG 11

11.1	$P(A) + P(B) = 0,52$ $0,4 + P(B) = 0,52$ $P(B) = 0,12$	✓ substitution ✓ answer (2)
11.2.1	$P(\text{sandwich}) = \frac{4}{25}$  <b>OR/OF</b> $0,02 + 0,01 + 0,04 + 0,09 = \frac{4}{25} = 0,16$	✓ answer (1)  <b>OR/OF</b> ✓ answer (1)
11.2.2	$P(\text{at least two events}) = 0,02 + 0,01 + 0,03 + 0,04$ $= 0,1$	✓ $0,02 + 0,01 + 0,03 + 0,04$ ✓ answer (2)
11.2.3	$P(\text{not any}) = 1 - (0,1 + 0,04 + 0,09 + 0,2)$ $= 0,57$	✓ $1 - (0,1 + 0,04 + 0,09 + 0,2)$ ✓ answer (2)
11.3.1	$7! = 5040$	✓ $7!$ (1)
11.3.2	$P(4 \text{ players alphabetically}) = \frac{1}{7 \times 6 \times 5 \times 4} = \frac{1}{840}$	✓ 1 ✓ 840 ✓ $\frac{1}{840}$ (3)

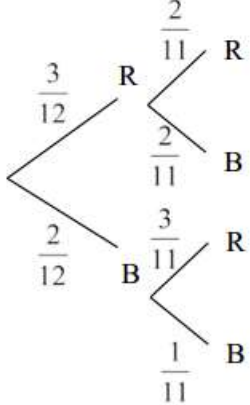
## Activity/Aktiwiteit 4

### QUESTION 10/VRAG 10

10.1.1	<p>Event A</p> <p>Event B</p> <p>HC HN SC SN</p>	<p>✓ Event A</p> <p>✓ Event B Medication: for <math>P(C) = \frac{3}{5}</math></p> <p>✓ Event B sugar pill: for <math>P(NC) = \frac{7}{10}</math></p> <p>(3)</p>
10.1.2	<p><math>P(\text{Not Cured}) = P(H) \times P(NC) + P(S) \times P(NC)</math></p> <p><math>= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)</math></p> <p><math>= \frac{11}{20} = 0,55</math></p>	<p>✓ substitution</p> <p>✓ answer (2)</p>
10.2.1	<p><math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math></p> <p><math>P(A \text{ and } B) = \frac{13}{20} - \frac{2}{5} - \frac{1}{4} = 0</math></p> <p>Events are mutually exclusive (Gebeurtenisse is onderling uitsluitend)</p> <p><b>OR/OF</b></p> <p><math>P(A) + P(B) = \frac{2}{5} + \frac{1}{4}</math></p> <p><math>= \frac{13}{20}</math></p> <p><math>P(A \text{ or } B) = P(A) + P(B)</math></p> <p><math>P(A \text{ and } B) = 0</math></p> <p>Events are mutually exclusive (Gebeurtenisse is onderling uitsluitend)</p>	<p>✓ substitution</p> <p>✓ answer (<math>P(A \text{ and } B) = 0</math>) (2)</p> <p><b>OR/OF</b></p> <p>✓ substitution</p> <p>✓ answer (<math>P(A \text{ and } B) = 0</math>) (2)</p>

## Activity/Aktiwiteit 5

### QUESTION/VRAAG 10

10.1	 <p> <math>P(\text{One Red and One Blue})</math>  <math>= P(\text{Red, Blue}) + P(\text{Blue, Red})</math>  <math>= \left(\frac{3}{12}\right) \times \left(\frac{2}{11}\right) + \left(\frac{2}{12}\right) \times \left(\frac{3}{11}\right)</math>  <math>= \frac{1}{11}</math> </p>	<p> <math>\checkmark \left(\frac{3}{12}\right) \times \left(\frac{2}{11}\right)</math>  <math>\checkmark \left(\frac{2}{12}\right) \times \left(\frac{3}{11}\right)</math>  <math>\checkmark</math> addition of products  <math>\checkmark</math> answer         </p> <p>(4)</p>
10.2.1	<p> <math>a = 0,48 \times 250</math>  <math>a = 120</math> </p>	<p> <math>\checkmark</math> answer         </p> <p>(1)</p>
10.2.2	<p> <math>b = 150</math>  <math>P(S) \times P(F)</math>  <math>= \frac{200}{250} \times \frac{150}{250}</math>  <math>= 0,48</math>  <math>= P(S \text{ and } F)</math>            These events are independent /  <i>Hierdie gebeurtenisse is onafhanklik</i> </p>	<p> <math>\checkmark b</math>  <math>\checkmark P(S) \times P(F)</math>  <math>\checkmark \frac{200}{250}</math> and <math>\frac{150}{250}</math>  <math>\checkmark</math> conclusion            (with realistic probabilities)         </p> <p>(4)</p>
		[9]



## Activity/Aktiwiteit 6

### QUESTION/VRAAG 2

2.1	$a = 73$ $b = 42$ $c = 107$ $d = 68$	$\checkmark a = 73$ $\checkmark b = 42$ $\checkmark c = 107$ $\checkmark d = 68$	(4)																
	<table border="1"> <thead> <tr> <th></th><th>Aged &lt; 40</th><th>Aged <math>\geq</math> 40</th><th>Totals</th></tr> </thead> <tbody> <tr> <td>Liked the movie</td><td>65</td><td>37</td><td>102</td></tr> <tr> <td>Did not like the movie</td><td><math>b = 42</math></td><td>31</td><td><math>a = 73</math></td></tr> <tr> <td>Totals</td><td><math>c = 107</math></td><td><math>d = 68</math></td><td>175</td></tr> </tbody> </table>		Aged < 40	Aged $\geq$ 40	Totals	Liked the movie	65	37	102	Did not like the movie	$b = 42$	31	$a = 73$	Totals	$c = 107$	$d = 68$	175		
	Aged < 40	Aged $\geq$ 40	Totals																
Liked the movie	65	37	102																
Did not like the movie	$b = 42$	31	$a = 73$																
Totals	$c = 107$	$d = 68$	175																
2.2	$P(\text{less than 40 and did not like the movie}) = \frac{42}{175} \quad (0,24)$	$\checkmark 42$ $\checkmark 175$	(2)																
2.3	$P(\text{less than 40 and liked the movie}) = \frac{65}{175} = 0,37 \quad (0,3714285714...)$ $P(\text{Age less than 40}) = \frac{107}{175}$ $P(\text{Critic liked the movie}) = \frac{102}{175}$ $P(\text{Age less than 40}) \times P(\text{Critic liked the movie}) = \frac{107}{175} \times \frac{102}{175} = 0,36 \quad (0,3563755102...)$ <p>Since <math>P(\text{less than 40 and liked the movie}) \neq P(\text{Age less than 40}) \times P(\text{Critic liked the movie})</math>, we can conclude that the events are not independent/nie onafhanklik nie.</p>	$\checkmark P(\text{less than 40 and liked the movie}) = \frac{65}{175}$ $\checkmark P(\text{Age less than 40}) = \frac{107}{175}$ <b>OR</b> $P(\text{Critic liked the movie}) = \frac{102}{175}$ $\checkmark P(\text{Age less than 40}) \times P(\text{Critic liked the movie}) = 0,36$ $\checkmark \text{conclusion}$	(4)																
	<b>OR</b> $P(\text{less than 40 and did not like the movie}) = \frac{42}{175} = 0,24$ $P(\text{Age less than 40}) = \frac{107}{175}$ $P(\text{Critic did not like the movie}) = \frac{73}{175}$ $P(\text{Age less than 40}) \times P(\text{Critic did not like the movie}) = \frac{107}{175} \times \frac{73}{175} = 0,255 \quad (0,2550530612...)$ <p>Since <math>P(\text{less than 40 and did not like the movie}) \neq P(\text{Age less than 40}) \times P(\text{Critic did not like the movie})</math>, we can conclude that the events are not independent/nie onafhanklik nie.</p>	$\checkmark P(\text{less than 40 and did not like the movie}) = \frac{42}{175}$ $\checkmark P(\text{Age less than 40}) = \frac{107}{175}$ <b>OR</b> $P(\text{Critic did not like the movie}) = \frac{73}{175}$ $\checkmark P(\text{Age less than 40}) \times P(\text{Critic did not like the movie}) = 0,255$ $\checkmark \text{conclusion}$	(4) [10]																



## BIBLIOGRAPHY/BIBLIOGRAFIE

1	DBE November 2008 – 2024 Marking Guidelines / <i>Nasienriglyne</i>
2	DBE Feb/Mar and June 2008 – 2024 Marking Guidelines / <i>Nasienriglyne</i>